Lidar Odometry Undistortion

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This note describes lidar point cloud undistortion, particularly in the context of KISS-ICP [1].

World frame is $\mathcal{F}_a$, LiDAR frame is $\mathcal{F}_l$. The LiDAR frame as particular time instant t_k is denoted $\mathcal{F}_{l_k}$. The map frame $\mathcal{F}_m$ is set to the first LiDAR frame $\mathcal{F}_{l_0}$.

KISS-ICP uses a constant velocity model, where velocity at time t_k is computed using the previous two time instants t_{k-2} and t_{k-1}. Formally, the following model is used,

$$\mathbf{C}_{ml_k} = \mathbf{C}_{ml_{k-1}} \operatorname{Exp}(\Delta t_{k,k-1} \boldsymbol{\omega}_{l_{k-1}}^{l_{k-1}m}) \tag{1}$$

$$\mathbf{r}_m^{l_k m} = \mathbf{r}_m^{l_{k-1}m} + \Delta t_{k,k-1} \mathbf{v}_m^{l_{k-1}m} \tag{2}$$

$$= \mathbf{r}_m^{l_{k-1}m} + \Delta t_{k,k-1} \mathbf{C}_{ml_{k-1}} \mathbf{v}_{l_{k-1}}^{l_{k-1}m} \tag{3}$$

where $\Delta t_{k,k-1} \triangleq t_k - t_{k-1}$ and $\mathbf{v}_m^{l_{k-1}m} \triangleq \mathbf{v}_m^{l_{k-1}m/m}$, the derivative frame is dropped for brevity. The constant velocity is resolved in the body frame.

Isolating for the velocities,

$$\boldsymbol{\omega}_{l_{k-1}}^{l_{k-1}m} = \frac{1}{\Delta t_{k,k-1}} \operatorname{Log}(\mathbf{C}_{ml_{k-1}}^{\mathsf{T}} \mathbf{C}_{ml_k}), \tag{4}$$

$$\mathbf{v}_{l_{k-1}}^{l_{k-1}m} = \frac{\mathbf{C}_{ml_{k-1}}^{\mathsf{T}} \left(\mathbf{r}_m^{l_k m} - \mathbf{r}_m^{l_{k-1}m} \right)}{\Delta t_{k,k-1}}. \tag{5}$$

For a constant velocity model, such that $\boldsymbol{\omega}_{l_k}^{l_k m} \approx \boldsymbol{\omega}_{l_{k-1}}^{l_{k-1}m}$ and $\mathbf{v}_m^{l_k m} \approx \mathbf{v}_m^{l_{k-1}m}$, the angular velocity may be expressed as

$$\boldsymbol{\omega}_{l_k}^{l_k m} = \frac{1}{\Delta t_{k-1,k-2}} \operatorname{Log}(\mathbf{C}_{ml_{k-2}}^{\mathsf{T}} \mathbf{C}_{ml_{k-1}}), \tag{6}$$

$$\mathbf{v}_{l_k}^{l_k m} = \frac{\mathbf{C}_{ml_{k-2}}^{\mathsf{T}} \left(\mathbf{r}_m^{l_{k-1}m} - \mathbf{r}_m^{l_{k-2}m} \right)}{\Delta t_{k-1,k-2}}. \tag{7}$$

At a time t, the LiDAR measures a relative point position $\mathbf{r}_{l(t)}^{p_i l(t)}$, where $\underrightarrow{p_i}$ is a point in the environment.

$$\mathbf{r}_{l(t)}^{p_i l(t)} = \mathbf{C}_{l(t) l_k} \mathbf{r}_{l_k}^{p_i l_k} + \mathbf{r}_{l(t)}^{l_k l(t)}. \tag{8}$$

Note that the LiDAR scan *starts* at t_{k-1}, and *finishes* at t_k. While this measurement model can be used directly, the time offsets and thus $\mathbf{T}_{l(t) l_k}$ are different for each point. Furthermore, for extracting features and downsampling, it is preferable to have points that all have the same timestamp. Therefore, the deskewed points at time k are computed as $\mathbf{r}_{l(t)}^{p_i l(t)}$, where $\underrightarrow{p_i}$ is a point in the environment,

$$\mathbf{r}_{l_k}^{p_i l_k} = \mathbf{C}_{l_k l(t)} \mathbf{r}_{l(t)}^{p_i l(t)} + \mathbf{r}_{l_k}^{l(t) l_k}. \tag{9}$$

Using the constant velocity assumption,

$$\mathbf{C}_{l_k l(t)} = \operatorname{Exp}((t_k - t)\boldsymbol{\omega}_{l_k}^{l_k m}), \tag{10}$$

$$\mathbf{r}_{l_k}^{l(t) l_k} = \mathbf{v}_{l_k}^{l_k m}(t_k - t). \tag{11}$$

Thus, the undistorted points are each written as

$$\mathbf{r}_{l_k}^{p_i l_k} = \operatorname{Exp}((t_k - t)\boldsymbol{\omega}_{l_k}^{l_k m}) \mathbf{r}_{l(t)}^{p_i l(t)} + \mathbf{v}_{l_k}^{l_k m}(t_k - t). \tag{12}$$

References

[1] I. Vizzo, T. Guadagnino, B. Mersch, L. Wiesmann, J. Behley, and C. Stachniss, "Kiss-icp: In defense of point-to-point icp – simple, accurate, and robust registration if done the right way," *IEEE Robotics and Automation Letters*, vol. 8, no. 2, pp. 1029–1036, 2023.